The data reduction for Lab #3 is outlined below.

When the system is in the choked flow condition (i.e. $P_{tank} > P_{star}$) the equation in the course notes (by Prof. Chen) on page 65 is valid:

$$t = \left[\frac{V_{tank}}{\left(\sqrt{k \cdot R \cdot T_o}\right) \cdot C_D \cdot A_{nozzle}}\right] \cdot \left[\frac{2}{k-1} \cdot \left(\frac{k+1}{2}\right)^{\frac{k+1}{2 \cdot (k-1)}}\right] \cdot \left[\left(\frac{P_o}{P_{tank}}\right)^{\frac{(k-1)}{2 \cdot k}} - 1\right] \qquad \text{Eq. (A)}$$

where

t = the time from the start of the blow-down process

C_D = the discharge coefficient

V_{tank} = the volume of the tank (230 in^3)

R = gas constant for air

k = specific heat ratio for air

A_{nozzle} = area of the nozzle (Diameters are: #65 -> 0.035", #70 -> 0.028",
#75 -> 0.021", #80 -> 0.0135"
long tube -> 1 mm)

T_o = starting temperature of process (K)

P_o = starting pressure of process

P_{tank} = tank pressure during process

The variables are presented in the example calculations below. See page 62-65 of the notes for the theoretical development.

The gas (air) properties are: $k := 1.4$ $\quad R := 287 \cdot \frac{J}{kg \cdot K}$

The measured atmospheric pressure is: $P_a := 14.9 \cdot psi$ $\quad P_a = 1.027319 \times 10^5 \, Pa$

From converging nozzle relationships, when the tank pressure is higher than the critical pressure, P_{star}, the flow rate is the choked mass flow rate. The critical pressure P_{star} can be determined from

$$\frac{P_a}{P_{star}} = \left(\frac{2}{k+1}\right)^{\frac{k}{k-1}} \quad \text{or} \quad P_{star} = P_a \cdot \left[\left[\frac{2}{(k+1)}\right]^{\left[\frac{k}{(k-1)}\right]}\right]^{-1}$$

since, for air $\left[\left[\frac{2}{(k+1)}\right]^{\frac{k}{(k-1)}}\right]^{-1} = 1.892929$ $\quad P_{star} = P_a \cdot 1.892929$

For the measured atmospheric pressure $\quad P_{star} := P_a \cdot \left[\left[\frac{2}{(k+1)}\right]^{\left[\frac{k}{(k-1)}\right]}\right]^{-1}$

The <u>critical pressure</u> is $\quad P_{star} = 28.204644\,\text{psi}$

The <u>measured temperature and pressure at the start of the blow-down process</u> are:

$T_o := (25.18 + 273.15) \cdot K \qquad T_o = 298.33$

$P_o := (57.37 + 1.158) \cdot psi \qquad P_o = 4.035364 \times 10^{5}\,kg\,m^{-1}\,s^{-2}$

<u>Tank and nozzle variables</u> are given:

$V_{tank} := 230 \cdot in^3 \qquad V_{tank} = 3.769025 \times 10^{-3}\,m^3 \qquad V_{tank} = 3.769025\,L$

Consider nozzle #65

$D_{nozzle} := 0.035 \cdot in \qquad D_{nozzle} = 8.89 \times 10^{-4}\,m$

$A_{nozzle} := \pi \cdot \frac{D_{nozzle}^{2}}{4} \qquad A_{nozzle} = 6.207167 \times 10^{-7}\,m^2 \qquad A_{nozzle} = 9.621128 \times 10^{-4}\,in^2$

<u>Time to unchoked flow</u>, where $P = P_{star}$ (call this t_{star}), it is really just an indicator of the end of choked flow. After this point in time the flow is no longer choked, and this assumption in the theoretical development is violated.

Procedure for finding $C_{D:}$

1) Use the measure pressure as "P_{star}" and the time from the start of the process as "t_{star}" in the P_{star} vs. t_{star} equation.

2) Using P_{star}, t_{star} (measured P and t) and the other variables, find C_D by solving the equation below for C_D

$$t_{star} = \left[\frac{V_{tank}}{\left(\sqrt{k\cdot R\cdot T_o}\right)\cdot C_D\cdot A_{nozzle}}\right]\cdot\left[\frac{2}{k-1}\cdot\left(\frac{k+1}{2}\right)^{\frac{k+1}{2\cdot(k-1)}}\right]\cdot\left[\left(\frac{P_o}{P_{star}}\right)^{\frac{(k-1)}{2\cdot k}} - 1\right]$$

Set $$Const := \frac{2}{k-1}\cdot\left(\frac{k+1}{2}\right)^{\frac{k+1}{2\cdot(k-1)}} \qquad Const = 8.64$$

Therefore

$$t_{star} = \left[\frac{V_{tank}}{\left(\sqrt{k\cdot R\cdot T_o}\right)\cdot C_D\cdot A_{nozzle}}\right]\cdot Const\cdot\left[\left(\frac{P_o}{P_{star}}\right)^{\frac{(k-1)}{2\cdot k}} - 1\right]$$

Solving for the discharge coefficient, C_D at $t_{star} := 21\cdot sec$

$$C_D := V_{tank}\cdot Const\cdot\frac{\left[\left(\frac{P_o}{P_{star}}\right)^{\left[\frac{1}{2}\cdot\frac{(k-1)}{k}\right]} - 1\right]}{\left[t_{star}\cdot\left(\sqrt{k\cdot R\cdot T_o}\cdot A_{nozzle}\right)\right]} \qquad C_D = 0.79315$$

Hint: note the values for some of the individual terms to help you troubleshoot your data reduction.

$$Const2 := \left[\left(\frac{P_o}{P_{star}}\right)^{\frac{(k-1)}{2\cdot k}} - 1\right] \qquad Const2 = 0.10992$$

$$\sqrt{k\cdot R\cdot T_o} = 346.221019\,m\,s^{-1} \qquad \frac{V_{tank}}{A_{nozzle}} = 6.072053\times 10^{3}\,m$$ for nozzle #65.

In general, expect CD to be from 0.6 to 0.9. Values greater than 1 and less than 0 are impossible. If the value is 1 there are no losses due to friction etc. The smaller the value the greater the losses.

For a good discussion of this see Zucrow and Hoffman (*Gas Dynamics*, Vol. 1, John Wiley and Sons, 1976, pp.187-200). There you will find the figure shown below, where the discharge coefficient for unchoked flow is shown to increase with increasing tank stagnation pressure up to the choked condition. As given by the theory, the C_D is constant (essentially according to the measurements) in the choked condition for the three nozzles considered.

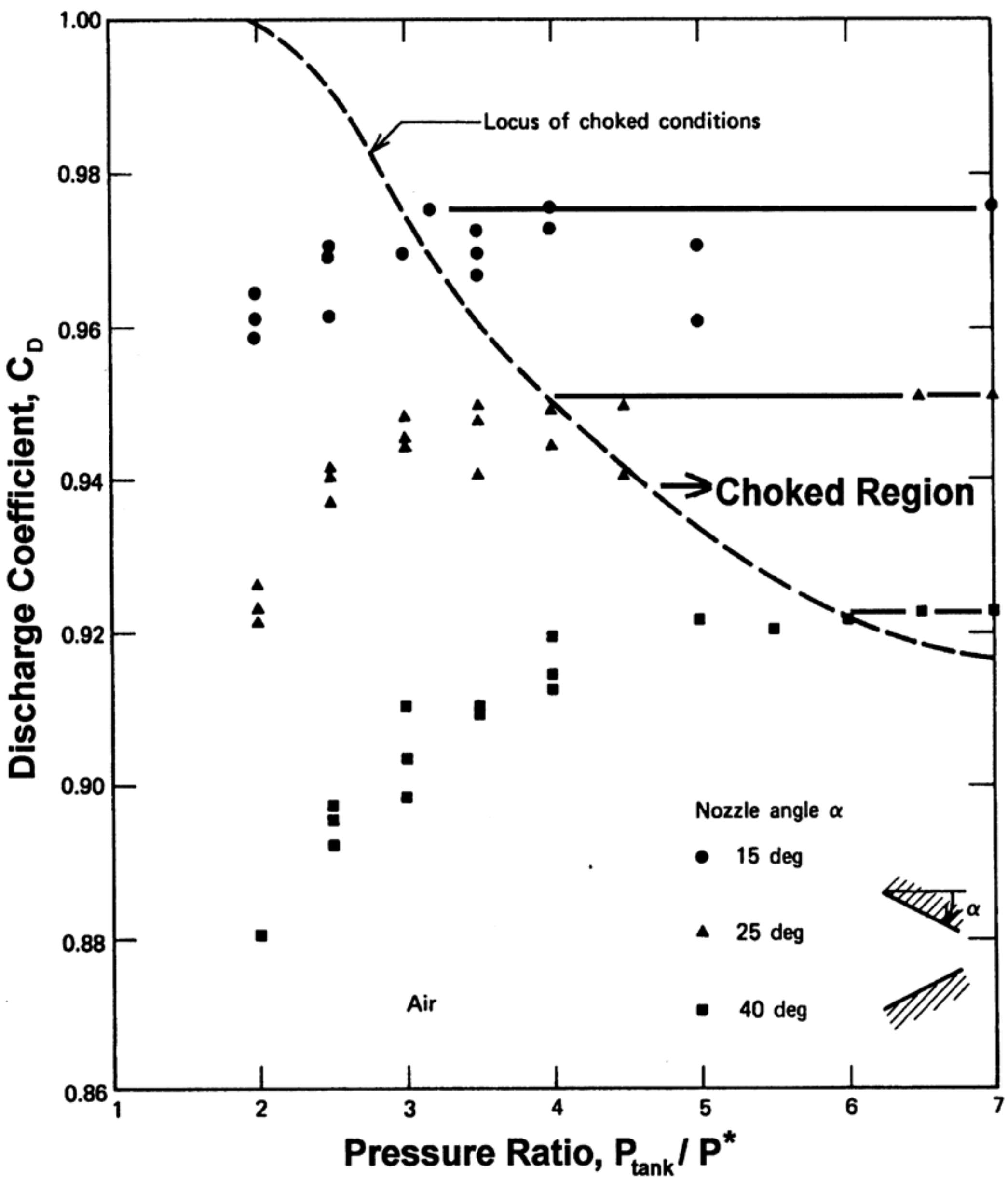

Figure 4.16. Experimental discharge coefficients for conical converging nozzles